# 2.5 Determinants & Inverses

## Determinants

- Each square matrix has a determinant

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \rightarrow \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

(matrix) (notation to find det.)

Ex

$$\begin{bmatrix} 0 & -2 \\ 8 & -6 \end{bmatrix} = \begin{vmatrix} 0 & -2 \\ 8 & -6 \end{vmatrix} = 0 \cdot -6 - 8 \cdot -2$$
$$= 0 + 16$$
$$= 16$$

**Inverse matrices** - Has the same concept as reciprocals $\left(\frac{1}{7} \rightarrow 7\right)$

Denoted $A^{-1}$

**Identity matrix** - A matrix of order one.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

($I$: Identity)

**Calculate Inverse**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

($ad - cb$: determinant)

Swap a & d, put negative in front of c & b; ÷ determinant

Ex

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{4 \cdot 6 - 2 \cdot 7} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$
$$= \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} .6 & -.7 \\ -.2 & .4 \end{bmatrix}$$

Check

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} .6 & -.7 \\ -.2 & .4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

#34

$$4x - y = 1$$
$$x + 2y = 7$$

$$\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

Find Inverse

$$\frac{1}{4 \cdot 2 - 1 \cdot -1} \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

$$\frac{1}{9} \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 27 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
$$(1, 3)$$